CONFINEMENT IN 3D GLUODYNAMICS AS A 2D CRITICAL PHENOMENON

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ABSTRACT

Gluodynamics in 3D spacetime with one spatial direction compactified into a circle of length L is studied. The confinement order parameters, such as the Polyakov loops, are analyzed in both the limits $L \to 0$ and $L \to \infty$. In the latter limit the behavior of the confinement order parameters is shown to be described by a 2D non-linear σ-model on the compact coset space $G/ad\ G$, where G is the gauge group and $ad\ G$ its adjoint action on G. Topological vortex-like excitations of the compact field variable cause a Kosterlitz-Thouless phase transition which is argued to be associated with the confinement phase transition in the 3D gluodynamics.

1. Introduction. In recent years there exists an increasing interest in relating some QCD non-perturbative phenomena to 2D field theories properties[2]. A way to describe a relation between 3D gluodynamics and its effective 2D field theories is to compactify one of the two *spatial* directions into a circle of length L. The length L plays the role of an interpolating parameter between the 2D and 3D gluodynamics ($L \to 0$ and $L \to \infty$, respectively). When L goes to zero, gauge fields homogeneous in the compactified coordinate will dominate because the non-homogeneous components of the gauge fields become massive (the mass is of order $1/L$). The effective theory in this limit is QCD_2 with the adjoint scalar matter[1,2,3].

Our interest lies on both the study of the relevant order parameters describing 3D confinement and constructing 2D models which can explain their critical behaviors. The sought effective 2D theory should therefore be valid beyond the small L regime. The main aim of the paper is to describe such a theory.

2. Characterization of QCD_3 confinement in 2D. One can define the transverse (along the compactified direction) and the longitudinal (along the unbounded directions) Polyakov loops as the *v.e.v.*'s of the 2D $SU(N)$ field operators:

$$\begin{aligned} P &\equiv \lim_{\beta\to\infty} \langle \mathrm{Tr} g(0)\rangle_\beta \quad , \quad g(x_1,x_2) = \exp[ig_3 \textstyle\int_0^\beta dx_0 A_0(x_0,x_1,x_2)] \in SU(N) \ ; \\ \bar{P}(L) &\equiv \langle \mathrm{Tr}\bar{g}(0)\rangle_L \quad , \quad \bar{g}(x_0,x_1) = \exp[ig_3\sqrt{L}\phi(x_0,x_1)] \in SU(N) \ ; \\ & \qquad\qquad\qquad\qquad \phi(x_0,x_1) \equiv L^{-1/2} \textstyle\int_0^L dx_2 A_2(x_0,x_1,x_2) \ . \end{aligned} \tag{1}$$

Since g and $\bar{g}$ are 2D operators, the behavior of the Polyakov loops can be inferred from pure 2D considerations.

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In the small L limit exact calculations of these loops are possible. This regime is characterized by $(P = 0, \bar{P} \neq 0)$. The other limit is expected to be given by $(P=0, \bar{P}=0)$. Therefore, although both regimes are in a confining phase $(P=0)$ they are not in the same *chiral* phase yet. The non-zero value of the transverse Polyakov loop $(\bar{P}\neq 0)$ indicates that the small L regime is in a chirality broken phase. When L grows, the timelike string tension (the one associated with P) stabilizes with L at the same critical length as $\bar{P}$ vanishes as lattice simulations show [4].

The physical interpretation of this result is very appealing. For small values of L, there is no difference in the confinement picture occurring in QED and QCD because color flux lines are forced to lie on the artificial strip of a small width L yielding a linearly rising potential between static charges. However the distribution of flux lines on the strip behaves very differently in both cases. In the abelian case flux lines spread out occupying the whole width independently of the strip size. In the non-abelian case the transverse gluon interactions increase the string tension value by keeping the flux lines more and more packed. When the critical length is reached the flux lines remains together independently of how much the strip width is enlarged. The fact that this phenomenon occurs at the same time as $\bar{P}$ vanishes tells us that we can use the transverse Polyakov loop to characterize the generation of the flux tube.

From a purely 2D point of view, the problem of the flux tube generation boils down in the understanding of the 2D field ϕ dynamics. Our strategy is to construct a consistent 2D model incorporating two important properties. Firstly, it must reduce to the known small width regime result which can be calculated exactly from QCD_3. Secondly, it has to exhibit a non-trivial 2D phase transition responsible for the vanishing of the *v.e.v.* of the 2D operator $\mathrm{Tr}\bar{g}$.

3. 2D effective theory. Our starting point is to perform the Fourier decomposition of the gauge field over the compactified coordinate x_2 and thereby to obtain an equivalent 2D theory. We choose the gauge $\partial_2 A_2 = 0$ or $A_2(x, x_2) = \phi(x)/\sqrt{L}$, in order for the theory to be invariant under gauge transformations depending only on the longitudinal coordinates (x_0, x_1). The gauge fixed action will be a function of the infinite set of the longitudinal modes $(\phi, a_\alpha, \{V_\alpha^n\})$. The non-zero mode fields $\{V_\alpha^n\}$ are massive, with bare masses proportional to $1/L$. The zero mode fields (ϕ, a_α) are classically massless.

An effective action for the ϕ field, which allows us to calculate $\langle \mathrm{Tr}\bar{g}\rangle$, is obtained by integrating out the non-zero modes. No matter how complicated the interaction is, the effective action has to possess two important properties. First of all, it has to enjoy the longitudinal gauge invariance for the 2D zero mode fields. The field a_α plays the role of the 2D gluon field, whereas ϕ is the adjoint scalar (matter) field. The second condition is that the ϕ field is not an ordinary Lie-algebra-valued field, rather it parametrizes the *compact* gauge group manifold G. Accordingly, the functional integration over the field ϕ involves a nontrivial local measure $d\phi M(\phi) = d\mu_H(\bar{g})$ with $d\mu_H$ being the left- and right-invariant Haar measure for $\bar{g}(x_0, x_1) \in G$. The origin of this dynamical condition on the field ϕ is easily understood if one calculates the Faddeev-Popov determinant in the gauge $A_2 = \phi/\sqrt{L}$, which coincides with the Haar

measure[5]. For instance for SU(2) it reads $M(\phi) = \sin^2(\epsilon|\phi|)/|\phi|^2$, where $|\phi| = \sqrt{\mathrm{Tr}\phi^2}$, $\epsilon = g_3\sqrt{L}/\sqrt{2}$, $\phi = \tau_a\phi_a/2$ and $\mathrm{Tr}\tau_a\tau_b = 2\delta_{ab}$ for the Pauli matrices.

When integrating out the gluon field a_α, we fix the unitary gauge $\phi = \tau_3\varphi/2$. The Faddeev-Popov determinant for this gauge is φ^2 so the final measure for the field φ assumes the form $\sin^2(\varphi\epsilon)$. The effective 2D action for the field φ should respect the discrete symmetry $T_n : \varphi \to \pm\varphi + 2\pi n\epsilon^{-1}$, (where n is an integer) which is known as the affine Weyl group W_A and can be viewed as a set of residual gauge transformations that are still allowed by the gauge $A_2 = \tau_3\varphi/2$ [5], that is, the physical values of φ lie in the interval $(0, \pi\epsilon^{-1})$ between two nearest zeros of the Faddeev-Popov determinant.

¿From the mathematical point of view, the effective 2D theory should be a non-linear σ-model on the coset space $G/ad\ G$, where $ad\ G$ is the adjoint action of the gauge group on the group manifold G. Note that the Lie-algebra-valued field ϕ parametrizes G and realizes the adjoint representation of the gauge group. Therefore its gauge non-equivalent configurations form the coset space $G/ad\ G$. To parametrize them, it is natural to choose a gauge where ϕ belongs to the Cartan subalgebra H. Then $G/ad\ G$ is isomorphic not to the whole Cartan subalgebra H, but to a compact domain in it H/W_A known as the Weyl cell[5]. In the $SU(2)$ case H is isomorphic to a line $I\!R$ and, hence, the Weyl cell is $(0, \pi\epsilon^{-1}) = I\!R/T_n$.

4. Flux tube formation as a Kosterlitz-Thouless transition. It is certainly not possible to find an explicit form of the effective action for the compact (cyclic) field variable φ. However, one can model its critical behavior. The field φ is compact so it should have topological vortex-like excitations that could cause the Kosterlitz-Thouless phase transition[6,7]. To describe the dynamics of vortices, the compact field φ has to be mapped on a non-compact field ϑ in the partition function path integral. To regularize the theory at short distances, we set the system on a 2D lattice. The variable $\epsilon\varphi_x \in (-\pi, \pi)$ at the lattice site x can be thought as an angular variable determining a direction of spin attached to x. That is, the effective theory describes a planar spin system. Applying a conventional technique[8] we get

$$\begin{aligned}
\mathcal{Z} &= \int_{-\pi/\epsilon}^{\pi/\epsilon} \prod_x \left(\epsilon d\varphi_x\, \sin^2(\varphi_x\epsilon)\right) \exp\left\{-\sum_{\langle x,x'\rangle} \mathcal{L}(\varphi_x, \varphi_{x'})\right\} &(2)\\
&= \sum_{[m_x]} \int_{-\infty}^{\infty} \prod_x (d\vartheta_x)\, \exp\left\{-\sum_{\langle x,x'\rangle} \left(\tilde{\mathcal{L}}_M(\vartheta_x, \vartheta_{x'}, \epsilon) + 2\pi i m_x\vartheta_x\right)\right\} &(3)
\end{aligned}$$

where m_x is an integer-valued field (vorticity at a site x), $\mathcal{L}(\varphi_x, \varphi_{x'})$ is a lattice Lagrangian of the cyclic field φ_x and $\langle x, x'\rangle$ specifies lattice sites involved in a pair-wise interaction.[†] The transformation (3) is the duality transformation that relates the strong coupling regime $\epsilon \to \infty$ of the model (2) to a weak coupling regime in (3).

In the Villain model[7,8], $\tilde{\mathcal{L}}_M$ is quadratic and the integral over the non-compact spin-wave field ϑ can be done so that the partition function assumes the form of that for the $2D$ Coulomb gas, where m_x plays the role of the charge distribution. The model (2) exhibits the Kosterlitz-thouless phase transition which, and it is our conjecture, can be associated with the stabilization of the flux tube at a certain critical width $L = L_c$. Indeed, the small L regime ($|\varphi_x| \ll \epsilon^{-1}$) corresponds to a low

[†] One may also assume $\mathcal{L}$ to contain non-local interactions, not just nearest-neighbor interactions.

temperature regime ($\epsilon^2 \to 0$) in (2). Below a certain critical temperature the spin system is in an "ordered phase"; spin-wave excitations are dominant, while vortices are bound in vortex-antivortex pairs. Above the critical temperature these pairs are expected to dissolve, which results in vanishing the *v.e.v.* $\langle Tr\, \bar{g} \rangle$.

To obtain a soluble (or analytically tractable) model, one should take into account two important features of our effective theory which are not present in the Kosterlitz-Thouless model and have the *gauge* origin. First, in addition to the local periodicity $\varphi_x \to \varphi_x + 2\pi\epsilon^{-1} n_x$, the Lagrangian $\mathcal{L}$ should respect a *local* $\mathbb{Z}_2$ symmetry $\varphi_x \to \pm\varphi_x$, the residual gauge transformation from the Weyl group (not to be confused with the transformations from the center $\mathbb{Z}_2$ of $SU(2)$, the later ones correspond to $\varphi_x \to \varphi_x + \pi$). Thus, our system is a planar spin system with an "extra" $\mathbb{Z}_2$-gauge group. In particular, this imposes a condition on the possible forms of $\tilde{\mathcal{L}}_M$: $\tilde{\mathcal{L}}_M(\vartheta_x, \vartheta_{x'}) = \tilde{\mathcal{L}}_M(-\vartheta_x, \vartheta_{x'}) = \tilde{\mathcal{L}}_M(\vartheta_x, -\vartheta_{x'})$. The second feature is the local measure $M = \prod_x \sin^2(\varphi_x \epsilon)$ (the Faddeev-Popov determinant). One can consider it as a crystalline magnetic field in the spin system: $\mathcal{L} \to \mathcal{L} + V_c$, $V_c = \gamma \ln M = -\gamma \sum_{n=1}^{\infty} n^{-1} (\cos 2\varphi_x \epsilon)^n$. Its effect is in a shift of the critical temperature[8].

5. Acknowledgments. A.J. would like to thank V. Branchina, M. Caselle, T. Heinzl and J. Polonyi for interesting discussions, and a conference grant from the European Union as well as a support of a doctoral fellowship from IVEI (Valencian Institution for Research and Studies, Spain) are gratefully acknowledged. A.F. has maintained illuminating conversations with S. Chandrasekharan, A.V. Matytsin and P. Zaugg. S.V.S. thanks the Department of Theoretical Physics of University of Valencia for the warm hospitality.

6. References

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